RESEARCH PAPER

Derivation of phenomenological turbulence theory in liquid with small additives of drag reducing agents

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ABSTRACT

Paper considers the issue of deriving the phenomenological turbulence theory in liquids treated with small additives of drag reducing agent. It also proposes the concept that for practical purposes, it is the phenomenological theory, which is relevant, since it determines the parameters of the phenomenon in question in the absence of detailed knowledge of the mechanisms of additives action, which, despite many years of intensive studies, remain either unknown or not fully understood. Different additives have different effects on shear turbulence in pipes and channels and, accordingly, change the integral characteristics of the turbulent flow in different ways. Some additives affect only the narrow wall-bounded areas of the flow without changing the turbulent viscosity in the flow core, while others act throughout the entire flow volume and significantly change the turbulent viscosity. Additives of the first type affect a turbulent flow by changing the boundary conditions in known models without changing the model coefficients. Additives of the second type change both the boundary conditions and the coefficients of the model itself. It is shown that the von Karman modified theory (model) of shear turbulence is equally suitable for describing the turbulent flow of a liquid with additives of the both first and second types. The universal drag equation with experimentally determined transfer coefficients that follows from this model enables calculating the hydraulic drag coefficient depending on the properties of the drag reducing agent used.

Key words: Pipeline, turbulent flow, hydraulic drag, drag reducing agent, phenomenological turbulence theory, turbulent viscosity, von Karman constant, universal equation, roughness.

INTRODUCTION

For more than 70 years of intensive studies of the Toms effect, discovered in 1948 and consisting in reducing the hydraulic drag of a turbulent fluid flow by adding a small amount of a special drag reducing agent, many different hypotheses have been put forward about the nature of this phenomenon and the mechanisms of its effect on turbulence in pipes, channels and boundary layers. Some of these hypotheses were later disproved by research results, others were confirmed in limited conditions, and some, although accepted by many researchers, still remain poorly understood. But most importantly, there is no well-established hydraulic theory, as well as unambiguously recognized formulas for calculating the hydraulic drag coefficient in turbulent flow with such additives.

It is very difficult or almost impossible to give a meaningful analysis of all theories and studies in the area of liquid turbulent flows with small additives of drag reducing agent (this is partially done in several well-known domestic and foreign works [1–7]), so we will focus only on the phenomenological aspect of this problem.

Hydraulic classification of drag reducing agents

The authors hereof keep to the opinion that one of the main reasons for absence of the generally accepted hydraulic theory and corresponding calculation formulas is the lack of classification of the additives in question. The general term 'small additive of drag reducer' is hiding a variety of different additives that demonstrate the Toms effect, but essentially differ from each other in the mechanisms of impact on turbulence.

One group of drag reducing agents (especially additives based on high-molecular polymers) impacts on turbulence only in the wall-bound layer of the pipe or channel. It is commonly believed that the molecules of such additives expand under the influence of high shear stresses, considerably increase their linear dimensions and therefore intensely affect the turbulent fluctuations generated by the walls (affect equally the fluctuations amplitude and frequency). Moreover, these additives practically do not work in the area of low shear velocities and low tangential stresses, in particular far from the walls, in the zone of the so-called turbulent flow core. Experiments show that drag reducing agent additives of this type are characterized by a high rate of destruction, where the polymer molecules break under the impact of turbulence, reducing the molecular weight and linear dimensions of the polymer and, as a result, losing the ability to reduce hydrodynamic drag.

Another group of drag reducing agents (various emulsions, surfactants, fine-dispersed suspensions, synthetic solutions, etc.) affect the turbulence in principally another way. Molecules of these additives are not expanded by shear stresses and affect not only the narrow wall-bounded layer, but the entire turbulent flow volume. It was proved experimentally that drag reducing agents of this type are either much less susceptible to destruction, or not destructed at all.

In hydro-mechanical terminology, the first type of drag reducing agents can be called surface-acting additives. By changing the structure of the turbulent flow only in a narrow wall-bounded layer, they affect the flow as a whole by changing the boundary conditions. In other words, surface-acting additives do not change the turbulent viscosity of the liquid. Additives of the second type act throughout the entire turbulent flow, that is, at each point of it, so they can be called volumetric additives. Volumetric additives change the turbulent viscosity of the liquid throughout the entire volume of the turbulent flow.

Turbulent viscosity

According to the hypothesis of J. V. Boussinesq, the tangential stress τ_{rx} in a turbulent flow is associated with the velocity $\dot{\gamma} = du/dr$ by the law

$$\frac{1}{\rho}\tau_{rx} = v_T\frac{du}{dr},$$

where $u(r)$ – the profile of averaged velocity in the pipe cross-section

r – radial coordinate (distance from the pipeline axis)

ρ – density

v_T – turbulent viscosity of the liquid.

This law looks similar to Newton's law of viscous friction, however, in contrast to it, the turbulent viscosity v_T is not an individual characteristic of the liquid but depends on the parameters of the turbulent flow. The problem is to understand what the turbulent viscosity depends on and how to calculate it.

Assuming that the turbulent viscosity depends only on the molecular viscosity v and the shear velocity $\dot{\gamma}$ $v_T = \Phi(v, du/dr)$, where Φ is the dimensionless function of it arguments), then it's impossible to get an expression for turbulent viscosity, because the formula lacks an argument with a linear parameter. Von Karman's revolutionary proposal was as follows: he proposed to consider the turbulent viscosity as dependent on the molecular viscosity and the first two derivatives ($u' = du/dr$ and $u'' = d^2u/dr^2$) of the radial profile of averaged flow velocities: $v_T = \Phi(v, u', u'')$. Ratio u', u'', in von Karman's opinion, was supposed to characterize the so-called length of the mixing path, introduced earlier by L. Prandtl. Because only one dimensionless combination $|u'|^3/(u'')^2$ can be formed from three dimensional values v, u', u'', the main von Karman's hypothesis can be represented as follows:

$$v_T = v\Phi\left(\frac{|u'|^3}{vu''^2}\right).$$

(1)

If one also takes into account the fact known from experiments that in a developed turbulent flow, the molecular viscosity of the liquid practically does not affect the core of the turbulent flow and manifests itself only in a relatively narrow layer near the inner surface of the pipeline, then the molecular viscosity v should be excluded from equation (1). Hence, it follows that the function Φ shall be linear in its argument, i.e., the turbulent viscosity is expressed as follows:

$$v_T = \kappa^2\left(\frac{|u'|^3}{u''^2}\right).$$

(2)

where the invariant coefficient κ is called the von Karman's constant (according to generally accepted experiments $\kappa = 0.4$). The existence of such a constant itself and its invariance with respect to flow modes indicate the validity of this approach.

Taking into account the expression (2) for the turbulent viscosity, the following equation is obtained:

$$\frac{1}{\rho}\tau_{rx} = \kappa^2\frac{|u'^3|}{u''^2}\frac{du}{dr} = -u_*^2\frac{r}{r_0}\quad\left(\frac{du}{dr} < 0\right),$$
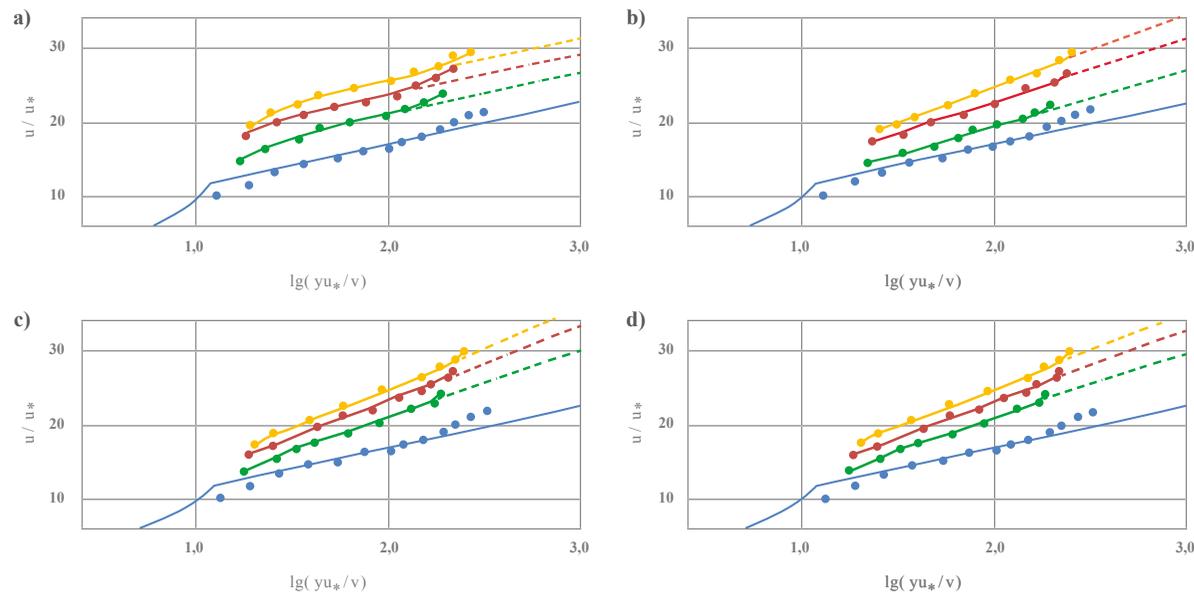
(3)

Figure 1. *Profiles of averaged turbulent flow velocity of water and solutions of drag reducing agents throughout the flow cross-section depending on Re number: • – water; • • • – additive solution (Re ≈ 7 – 15 · 103); a) polyacrylamide; b) surfactant (emulsion CB-102); c) detalan suspension; d) asbestos suspensions.*

where $u_* = \sqrt{\tau_w / \rho}$ – dynamic velocity

$\tau_w \equiv \left| \tau_{rx}(r_0) \right|$ – tangential stress on the pipe wall

r_0 – pipe radius.

Equation (3) is a so-called semi-empirical, and in fact is the phenomenological von Karman model for calculating the shear turbulence of the viscous incompressible liquid in a round tube

Then, a natural question emerges: is the von Karman constant changing with drag reducing agent additive?

There is still no agreement on this issue. For example, the monograph [7] states: 'On the one hand, it was found in [8] using nuclear magnetic resonance technique that the polyacrylamide additive in a water flow affects turbulence only in the wall-bounded region and does not change the von Karman constant in the region of the logarithmic velocity distribution law. On the other hand, authors [9], based on experiments with water containing small polyox additives, assert that the von Karman constant changed from κ = 0.4 at a concentration of θ = 0 to κ = 0.102 at θ = 50 g/t'.

In the authors' opinion, there can be no decisive answer to this question, since both options are possible: both a change and its absence. If we accept the results of the experiments of these authors, the polyacrylamide belongs to surface-acting drag reducing agents, and polyox – to volumetric additives.

In fact, this is the opinion expressed by L. Sedov and his colleagues, who have summarized experiments with drag reducing agents of various types [10]. Fig. 1 shows in

logarithmic coordinates η = yu_* / v (y – distance to the wall) the dimensionless profiles $u(\eta) = u / u_*$ of averaged turbulent flow velocity of water, treated with various drag reducing agents. [10, 11].

Fig. 1a shows that when the Re number increases, the averaged velocity profiles are almost parallel to each other outside the wall-bounded layer, which means their good correlation with the logarithmic profile in the flow core at the invariant von Karman constant. Fig. 1b to Fig. 1d show that the averaged velocity profiles change with Re number; this shall indicate the change in the slope of the velocities' logarithmic profile curve, which for liquids without drag reducing agents is written as $u(\eta) / u_* = \kappa^{-1} \lg \eta + B$ (B – const.), and, as a result – the change in the von Karman constant.

Let's give the intermediate deductions from these studies:

- the measurement data on flow velocities' turbulent profiles of water with 0.05 percent (500 ppm) polyacrylamide solution (polymer additive material) acquired using nuclear magnetic resonance are in good agreement with the known results obtained by other methods: polymer additives affect turbulence only in the wall-bounded area of the flow and do not change the von Karman constant (= 0.4) in the logarithmic profile area;

- similar measurements data on flow velocities' turbulent profiles of water-surfactant emulsions (emulsion SV-102, concentration 0.6% (60,000 ppm) showed that the introduction of surfactant changes the laws of turbulent exchange in the flow core (the von Karman constant in this case is less

than 0.4);

- additives of detalan (a water-alkaline base detergent), as well as additives made of asbestos fibers, affect turbulence both in the wall-bounded and in the core areas of the flow.

Thus, it can be concluded that additives of different nature have different effects on the areas of turbulent flow, and the mechanisms of this influence are also different. This is especially true, when this entails low, but still significant concentrations of additives (≈ 0,2–0,5 % or 20 000–50 000 ppm).

So, in general, the von Karman constant should be considered to be dependent on the concentration of drag reducing agent.

However, majority of the Toms effect researchers agree that it most often occurs in a relatively thin wall-bounded layer of the turbulent flow, where, in fact, turbulence originates and 'does not have any significant effect on the developed turbulence, that is, on the core of the flow' [7]. This is completely true for surface-acting drag reducing agents. Then, the question arises: how to take into account in mathematical models of shear turbulence, for example, in model (3), that the drag reducing agent affects the flow current only in a relatively thin wall-bounded layer?

Boundary conditions

By introducing the second derivative of the velocity profile along the radius into the turbulent flow model, von Karman increased the order of the differential equation in comparison with classic Navier-Stokes equations for the laminar liquid flow, hence follows the need for another additional boundary condition. In order to harmonize his model with the experiments, von Karman himself rejected the condition of 'sticking' $u(r_0) = 0$ to the walls of pipes or channels and assumed that $u(r) \rightarrow -\infty$ for $r \rightarrow r_0 (\eta \rightarrow 0)$. This approach enabled to take into account that the averaged velocity profile near the wall in a turbulent flow is very steep, but the model began to have a drawback. To eliminate it, many authors introduced one or more transition layers near the wall, endowing them with viscous, viscoelastic, or even plastic properties, and then conjugated the distributions in these layers with a logarithmic profile using empirical coefficients (for example, [12-17]).

The drawback of von Karman model can be eliminated if other boundary conditions are formulated in such a way as to reflect the presence of a drag reducing agent additive in case of its introduction into the liquid. That is why, the following problem statement appears to be natural: the von Karman model needs to be extended to the entire area of the turbulent flow up to the walls, and the changes occurring under the additive action in a relatively narrow wall-bounded layer shall be taken into account by the boundary conditions to equation (3) of this model. Since the solution of equation (3) has the form

$$u(r) = \frac{u_*}{\kappa}\left[\sqrt{\frac{r}{r_0}} + C_1 + C_2 \ln\left| \sqrt{\frac{r}{r_0}} - C_2 \right| \right], \quad (4)$$

where C_1 and C_2 –integrations constants defined from boundary conditions on the pipe walls (that is, for $r = r_0$), then two such boundary conditions are needed to determine these constants.

First boundary condition is the traditional 'sticking' condition, according to which

$$u_w \equiv u(r_0) = 0, \quad (5)$$

i.e., the liquid velocity on the inner surface of the pipeline is 0, both with and without drag reducing agent.

Second boundary condition, which is not available in classical theory, is obtained from considerations of dimension as a relationship of the form $\psi\left(u'_w, u''_w, v, \Delta\right) = 0$ between the first and second derivatives of the turbulent flow averaged velocity calculated on the wall, the molecular viscosity (which, unlike the flow core, is extremely significant near the wall), and the absolute roughness of the wall, Δ. From four arguments of the function Ψ, not just one, but two dimensionless complexes may be composed. Authors [18, 19] showed that this boundary condition may be written as follows:

$$v\frac{du}{dy}\bigg|_w = \frac{\kappa_1 u_*^2}{1 + f\left(\Delta u_* / v\right)} \quad \left(u_*^2 = \kappa^2 \frac{\left(u'_w\right)^4}{\left(u''_w\right)^2} \right), \quad (6)$$

where

κ_1 – an invariant constant (as von Karman constant, not depending upon the flow mode

$f\left(\Delta \cdot u_* / v\right)$ – some invariant function that reflects the effect of roughness Δ to form an average velocity profile (for a smooth pipe, $\Delta \approx 0$, $f \equiv 0$).

Universal drag equation

Taking into account the boundary conditions (5) and (6), the profile (4) of the turbulent flow averaged velocities takes the form:

$$u(r) = \frac{u_*}{\kappa}\left[\sqrt{\frac{r}{r_0}} - 1 + C_2 \ln\left| \frac{\sqrt{r / r_0} - C_2}{1 - C_2} \right| \right],$$

where $C_2 = \frac{1 + f\left(\Delta u_* / v\right)}{\kappa\kappa_1 u_* 2 r_0 / v}$. \quad (7)

Regarding the well-known formula of hydraulics:

$$\frac{\pi d^2}{4}\Delta p = \pi d\tau_w, \Delta p = \lambda\frac{\Delta x}{d}\cdot\frac{\rho u_{av}^2}{2}, \frac{1}{\rho}\tau_w = u_*^2,$$

where $d = 2r_0$ – internal diameter of the pipeline

u_{av} – average flow velocity,

we find that $\dfrac{u_*}{u_{av}} = \sqrt{\dfrac{\lambda}{8}}$.

Multiplying both parts of (7) by $2\pi r$ and integrating the result obtained along the radius from 0 to r_0, we obtain the universal drag equation, i.e. the equation for deriving the coefficient $\lambda(\mathrm{Re}, \varepsilon)$ of the hydraulic drag:

$$\sqrt{\frac{8}{\lambda}} = \frac{1}{\kappa}\left[\ln\frac{\kappa\kappa_1\mathrm{Re}\sqrt{\lambda/8}}{1+f\left(\varepsilon\mathrm{Re}\sqrt{\lambda/8}\right)} - 2,283\right], \tag{8}$$

which also takes into account that $u_{av}\cdot\pi d^2/4$ means the liquid flow rate in the pipe; $\mathrm{Re} = u_{av}\cdot d/v; \varepsilon = \Delta/d$.

If we take $\kappa = 0,4$, $\kappa_1 = 28$ and neglect the roughness ε of the inner surface of the pipeline, then equation (8) is converted into the well-known turbulence theory equation for determining the coefficient λ in hydraulically smooth pipes:

$$\frac{1}{\sqrt{\lambda}} = 0,884\ln\left(\mathrm{Re}\sqrt{\lambda}\right) - 0,8. \tag{9}$$

The explicit solution of this equation is fairly well approximated by the well-known Blasius formula: $\lambda = 0,3164/\mathrm{Re}^{0,25}$.

Function $f\left(\Delta u_*/v\right) \equiv f\left(\varepsilon\mathrm{Re}\sqrt{\lambda/8}\right)$, entering in the boundary condition (6) and in the universal drag equation (8), considers the roughness of the pipe or channel wall. In the phenomenological theory, it can be found from experiments, for example, by comparison with the well-known plots of I. Nikuradze.

Fig. 2 shows the function $Y \equiv f\left(X\right)$ versus the argument $X = \varepsilon\mathrm{Re}\sqrt{\lambda/8}$, found by comparing the equation (8) solutions with the results of I. Nikuradze experiments. One can see that up to a certain value of the argument $\left(X_* \approx 4,7-5,1\right)$, values of $f\left(X\right)$ are low (approximately equal to 0), but at $X_* > X$ they increas in a linear way [20]. Obviously, case $f\left(X\right) \approx 0$ corresponds to hydraulically smooth pipes, for which equation (9) is valid, i.e. hydraulic drag coefficient λ does not depend practically on the roughness of the pipeline inner surface, but is determined only by Re number, and may be represented explicitly by the Blasius formula. At $X_* > X$, function $f\left(X\right) > 0$, i.e. roughness cannot be ignored.
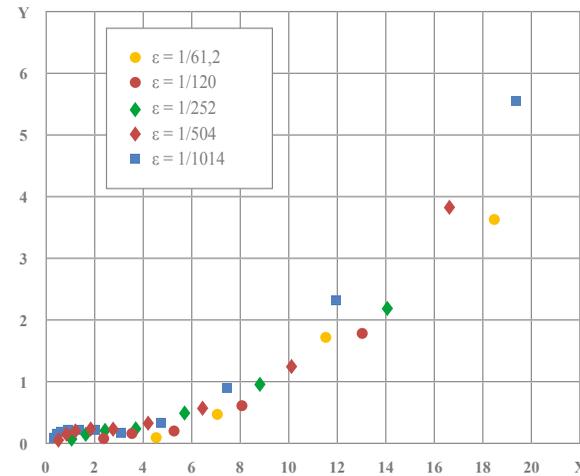
In some cases, piecewise linear approximation may be used

Figure 2. *Plot of function $f\left(X\right)$ versus $X = \varepsilon\mathrm{Re}\sqrt{\lambda/8}$.*

for the function $f\left(\varepsilon\mathrm{Re}\sqrt{\lambda/8}\right)$ [20]:

$$f\left(\varepsilon\mathrm{Re}\sqrt{\frac{\lambda}{8}}\right) = \begin{cases} \approx 0, if\ \varepsilon\mathrm{Re}\sqrt{\dfrac{\lambda}{8}} \le 4,9; \\ 0,31\left(\varepsilon\mathrm{Re}\sqrt{\dfrac{\lambda}{8}} - 4,9\right), if\ \varepsilon\mathrm{Re}\sqrt{\dfrac{\lambda}{8}} > 4,9 \end{cases}.$$

Generalization to the case of drag reducing agent use

Taking into account the boundary conditions, introducing a small additive of drag reducing agent into the turbulent flow affects, first of all, the changes in the values of invariant constants $\ae$, κ_1, and also in the invariant function $f\left(\Delta u_*/v\right) \equiv f\left(\varepsilon\mathrm{Re}\sqrt{\lambda/8}\right)$. Regardless of the physical mechanism of a particular additive effect on turbulence, these invariant values become functions of the concentration θ of this additive, i.e. $\kappa = \kappa\left(\theta\right)$; only for the volumetric drag reducing agents, $\kappa = \kappa_1\left(\theta\right)$ and $f\left(\Delta\cdot u_*/v,\theta\right)$. It is impossible to find the changed values of these invariants that do not depend on the geometry of the turbulent flow area, flow modes, etc., using theoretical approach, especially for any additives; one needs to use test experiments.

For binomial relationship $f = f\left(\Delta\cdot u_*/v,\theta\right)$, one may write:

$$f\left(\varepsilon\mathrm{Re}\sqrt{\frac{\lambda}{8}},\theta\right) =$$

$$= \begin{cases} \approx 0, if\ \varepsilon\mathrm{Re}\sqrt{\dfrac{\lambda}{8}} \le \kappa_3\left(\theta\right); \\ \kappa_2\left(\theta\right)\left(\varepsilon\mathrm{Re}\sqrt{\dfrac{\lambda}{8}} - \kappa_3\left(\theta\right)\right), if\ \varepsilon\mathrm{Re}\sqrt{\dfrac{\lambda}{8}} > \kappa_3\left(\theta\right) \end{cases}, \tag{10}$$
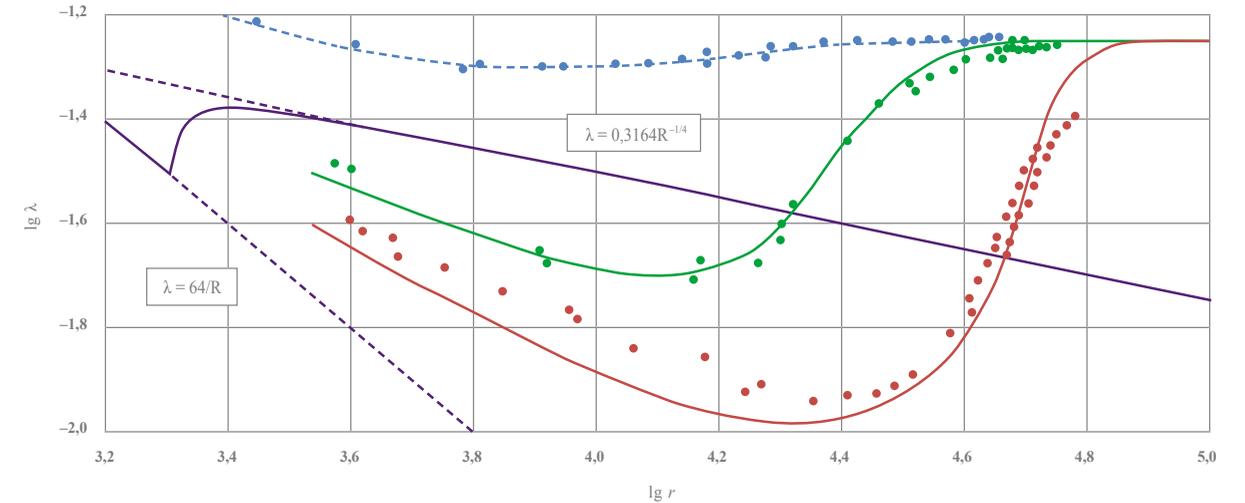
Figure 3. *The effect of reducing hydraulic drag coefficient λ at various liquids flow in a rough pipe: $\bullet$ – clean water ($v = 0$); $\bullet$ – polyethylene oxide solution (WSR-301; $M_{mol} \approx 4\cdot 10^6$) $v = 10$ ppm; $\bullet$ – polyethylene oxide solution (WSR-301; $M_{mol} \approx 4\cdot 10^6$) $v = 100$ ppm.*

then functions $\kappa(\theta)$, $\kappa_1(\theta)$, $\kappa_2(\theta)$ and $\kappa_3(\theta)$ shall be determined experimentally, where $\kappa(\theta) = 0,4$; $\kappa_1(0) = 28$; $\kappa_2(0) \approx 31$; $\kappa_3(0) \approx 4,9$.

Let's consider the issue of the coefficient $\kappa_1(\theta)$, which reflects the interaction of smooth walls with a turbulent flow, dependence on concentration θ of drag reducing agents. Experiments [13] showed that $\kappa_1(\theta)$ grows with increased concentration of practically any additive. If we refer to the boundary condition (6), which reflects the interaction of a turbulent flow with walls of a pipeline or channel, provided that their roughness is ignored, then it takes the form:

$$vu'_w = -\kappa_1\left(\theta\right)u_*^2.$$

If we introduce the dimensionless distance η to the wall according to equality $\eta = \left(r_0 - r\right)u_*/v$ and dimensionless velocity $u\left(\eta\right)/u_*$, then the boundary condition for the smooth wall gives:

$$v\frac{du}{dr} = v\frac{du}{d\eta}\cdot\frac{d\eta}{dr} = u_*\frac{du}{d\eta} = -\kappa_1\left(\theta\right)u_*^2$$

or $\dfrac{d}{d\eta}\left(\dfrac{u}{u_*}\right)\bigg|_w = -\kappa_1\left(\theta\right).$

Since at drag reducing agents being added to the turbulent flow, the coefficient κ_1, equal to ≈ 28 for liquid, increases, this means that *the steepness of the turbulent flow velocity profile near the pipe or channel walls also increases*, while the increase in the profile steepness depends on the concentration θ of the additive.

Let's consider the issue of the coefficient $\kappa_3(\theta)$, which characterizes the boundary between smooth and rough friction, dependence on concentration θ of drag reducing

agents. If $\varepsilon\mathrm{Re}\sqrt{\lambda/8} \le \kappa_3\left(\theta\right)$, then flow occurs in the region of smooth friction, otherwise, at $\varepsilon\mathrm{Re}\sqrt{\lambda/8} > \kappa_3\left(\theta\right)$, wall roughness starts to make its effect. Based on the experimental data available in the literature, it can be concluded that the drag reducing agent in a turbulent fluid flow expands the region of smooth flow, that is, shifts it towards large Re numbers. This is understandable: the agent additive decreases the hydraulic drag coefficient λ, so the product $\varepsilon\mathrm{Re}\sqrt{\lambda/8}$ attains the critical value of $\kappa_3(\theta)$ at large Re numbers.

Thus, Fig. 3 shows the results of hydrodynamic studies by scientists of the Moscow scientific school under the leadership of academician L.I. Sedov [8–12]. It follows from this figure that for clean water, the flow occurs in the region of mixed friction in the entire range under consideration (quite high ε value); for a polyethylene oxide solution of $\theta = 10$ ppm, the region of smooth friction ends at Re $\approx 16\,000$ (in Fig. 3 $R \equiv \mathrm{Re}$); for a polyethylene oxide solution of $\theta = 100$ ppm, the region of smooth friction ends already at Re $\approx 25\,000$, while at Re $> 63\,000$ (lg Re > 4.8) the effect of additive almost disappears. The authors [8] indicate: '*Near a rough wall with sufficiently high Re numbers, the effect of drag reduction by polymer additives disappears completely. With an increase in the solution concentration, other conditions being equal, this occurs at large Re number values*'.

Another important fact follows from the curves shown in Fig. 3. If we accept the statement that for each θ value at very large Re numbers $\left(\mathrm{Re} \to \infty, \mathrm{Re}\sqrt{\lambda/8} \to \infty\right)$, the effect of additive on λ disappears, then it follows from drag equation (8) with function f, defined by the formula (10), that at $\varepsilon\mathrm{Re}\sqrt{\lambda/8} > \kappa_3\left(\theta\right)$ the following condition must be met:

$$\sqrt{\frac{8}{\lambda}} = \frac{1}{\kappa}\left[\ln\frac{\kappa\kappa_1\theta}{\kappa_2\left(\theta\right)\varepsilon} - 2,283\right]. \tag{11}$$

From this, one can conclude that the ratio $\kappa_1(\theta)/\kappa_2(\vartheta)$ shall be maintained for all concentrations: $\kappa_2(\theta) \approx 0{,}011\,\kappa_1(\theta)$. If $\varepsilon\,\mathrm{Re}\sqrt{\lambda/8} > \kappa_3(\theta)$, the fact that this ratio remains constant does not affect the λ coefficient values (the flow occurs in the region of hydraulically smooth pipes), however, if this condition is violated (at transition into the region of rough pipes), gradual decrease in the effect of drag reducing agent occurs, and with very large Re numbers, the specified effect disappears completely. However, it should be noted that this conclusion is valid only for surface-acting drag reducing agents, i.e., additives with von Karman constant κ independent of concentration θ. If volumetric additive is considered, then $\kappa = \kappa(\theta)$, therefore the effect of the hydraulic drag reduction should take place even at very high Re numbers. The value of such effect is determined according to (11) by relative roughness ε, and by dependence of the model coefficients (first of all, of von Karman constant) on the additive concentration.

Thus, the universal drag equation has the form:

$$\sqrt{\frac{8}{\lambda}} = \frac{1}{\kappa(\theta)}\left[\ln\frac{\kappa(\theta)\kappa_1(\theta)\mathrm{Re}\sqrt{\lambda/8}}{1+f\left(\varepsilon\,\mathrm{Re}\sqrt{\lambda/8},\theta\right)} - 2{,}283\right], \qquad (12)$$

where the invariant function $f\left(\varepsilon\,\mathrm{Re}\sqrt{\lambda/8},\theta\right)$ is defined by the expression (10).

Equation (12) enables calculating the values of hydraulic drag coefficient $\lambda(\mathrm{Re},\varepsilon,\theta)$ using known (acquired in test experiments) relationships between coefficients $\kappa(\theta)$, $\kappa_1(\theta)$, $\kappa_3(\theta)$ and concentration θ of drag reducing agent.

Findings

Multiple drag reducing agent additives in terms of their effect on the turbulence may be divided into two types: surface-acting and volumetric.

Surface-acting additives change the flow structure in a narrow wall-bounded area only and do not affect the in the volume of the flow. When such additives are introduced into a liquid, von Karman constant does not change ($\kappa = 0{,}4$). Effect of the surface-acting additives on the turbulent flow occurs through boundary conditions in the flow margins. Contrary, the volumetric additives are manifested in every point of the flow and because of that change the turbulent liquid viscosity in the entire volume. The volumetric additives do change von Karman constant, it becomes dependent on their concentration ($\kappa < 0{,}4$).

Phenomenological model of the turbulent flow based on the known von Karman equation, supplemented by boundary conditions on the pipe walls, is not only an acceptable, but also quite reliable base for description of the liquid flow with small additives of drag reducing agents of both types – surface-acting and volumetric. First of all, the model is applicable for calculating the hydraulic drag coefficient in the liquid turbulent flow under conditions of lacking the detailed knowledge of additives' action mechanisms, which until now remain either unknown or not completely studied.

Competing interests

The authors declare that there is no competing interest regarding the publication of this paper.

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