RESEARCH PAPER

Numerical method for identifying the flow model in the line pipe

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ABSTRACT

With high availability of measuring tools and wide opportunities of modern computer technology, the existing methods of predictive estimations of hydraulic parameters for the fluids' pipeline transport seem to be too approximate. Due to this, it is relevant to adapt the most accurate relationships available in the scientific and technical literature to real conditions. Based on the review of analytical solutions for calculating friction losses in the pressure lines, the structure of relationships most accurately reflecting the experimental data of I. Nikuradze is determined, where the hydraulic drag coefficient λ is described by the piecewise-continuous relations, given by O. M. Ayvazyan. The hydraulic drag coefficient structural relationship shall be selected with the highest capability to summarize the experimental data available in the scientific and technical literature. Using the pressure measurement data, free parameters included in the selected relationship for the hydraulic drag coefficient shall be identified. The numerical computation algorithm is proposed that enables to recover the values of parameters in the structural relationship of hydraulic drag coefficient λ through multiple application of the well-known method of sensitivity functions and pressure measurement data in the line pipe. The procedure is described for generating the computing system of ordinary differential equations that enables for every fixed set of experimental data (pressure and flow rate) to determine (or correct, if necessary) the corresponding parameters in the unified structural relationship for hydraulic drag coefficient λ. The feature of the proposed algorithm is the absence of embedded cycles. Dynamic control of variable parameters in the hydraulic drag coefficient λ based upon the proposed approach enables to improve the predictive estimations accuracy of flow parameters while pumping fluids and to acquire additional data on the state of the fluids filling the inner pipeline space.

Key words: Hydraulic drag coefficient, sensitivity function, root-mean-square deviation, minimizing the functional.

INTRODUCTION

Since ancient times, the key hydrodynamic problem was to find out the essential nature of interaction between a moving solid and surrounding environment. The principles of the fluids drag and understanding the quantitative laws of hydraulic drag presented impenetrable barriers for a long time. The development of theoretical fundamentals of the viscous fluid theory was completed in 1845 with the formulation of the Navier-Stokes equations.

At the same time (1840–1846), theoretical and experimental studies of hydraulic drag in pipes and channels during laminar flow of viscous liquids were conducted [1], and the laminar flow instability and its transition to a new form of flow were revealed (Stokes, 1843). Wide application of these equations for solving problems of a viscous liquid flow was hindered by serious mathematical difficulties in finding their analytical solutions [1]. Reynolds for the first time obtained a system (non-closed) of differential equations for incompressible fluid turbulent flow. For 120 years (after Reynolds work publication in 1898 [2]), attempts by the world's leading scientists to create a closed system of equations for the theory of turbulence have been unsuccessful.

The most productive for the present is the phenomenological approach, which is based on postulating the link of turbulence with characteristics of the averaged flow. In the semi-empirical theory of turbulence [3], this link option is implemented by Prandtl in the form of a simple relationship between the velocity pulsations u'_r and the gradient of average velocity $\bar{u}$:

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$$u'_z = l\frac{d\bar{u}}{dr}, \tag{1}$$

where l – length of the 'mixing' path.

Using (1), Prandtl finally obtained so called logarithmic velocity profile, the applicability of which for the entire flow volume was confirmed by independent measurement of hydraulic drag and distribution of axial velocities in a wide range of Reynolds numbers $4\cdot10^3 \le \mathrm{Re} \le 3{,}2\cdot10^6$ in experimental studies of I. Nikuradze [4, 5]:

in hydraulically smooth pipe

$$\frac{\bar{u}}{u_*} = \frac{1}{\kappa}\ln\left[\frac{u_*\left(R_0 - r\right)}{\nu}\right] + C_r, \tag{2}$$

in hydraulically rough pipe

$$\frac{\bar{u}}{u_*} = \frac{1}{\kappa}\ln\left(\frac{R_0 - r}{k_s}\right) + C_{sh}. \tag{3}$$

Velocity distributions (2), (3) at $\kappa = 0.4$, $C_r = 5.5$ and $C_{sh} = 8.48$. lead to the same relationships for the hydraulic drag coefficient λ, as direct measurements of pressure loss:

for hydraulically smooth pipes

$$\frac{1}{\sqrt{\lambda_r}} = 2\lg\left(\mathrm{Re}\sqrt{\lambda_r}\right) - 0{,}8, \tag{4}$$

for hydraulically rough pipes

$$\frac{1}{\sqrt{\lambda_{sh}}} = 2\lg\left(\frac{R_0}{k_s}\right) + 1{,}74. \tag{5}$$

However, one should keep in mind the debatable nature of Prandtl's assumptions given below, which allowed him to integrate in a quite simple way the equation that bears his name.

The reasoned doubts about the validity of the logarithmic distribution of the longitudinal velocity over the entire flow cross-section were expressed by many researchers, for example in papers [6-8]. For the first time, the increasing deviation between experimentally measured velocities and logarithmic distributions was indicated in [9]. A number of attempts made to refine the velocity field are discussed in [10]. Particularly noteworthy is the work of A. A. Satkevich [11], which notes the logical and mathematical shortcomings of the Prandtl – Karman theory. Introduction of a correction factor that takes into account the flow 'constraint' in the pipe, enabled the author of the cited work to eliminate one of the theory 'alogisms', as well as to exclude the need to refer to a 'lucky chance' (a special stroke of luck), mentioned by Prandtl [12].

The thorough analysis of I. Nikuradze's experimental data, undertaken at various times by a number of authors [10, 13–16], confirmed that the logarithmic profile really best agrees with the data of I. Nikuradze's measurements only in a certain limited area of the flow, which is sometimes called the 'Prandtl layer' [17]. As far as new experiments were conducted, scientifically based corrections were made to the basic (logarithmic) formula. They mainly concerned the type of formula for the Prandtl length of the mixing path. The hypothesis on the universality of the law of averaged velocities distribution continued to cause attempts to improve it. The works of this orientation include G. I. Barenblatt studies on the theory of incomplete self-similarity, where an assumption is made that there is a layer with a constant turbulent shear stress in the wall-bound area [18, 19]. Some experimental studies revealed the existence of zones with negative turbulent viscosity. Taking this into account, a five-layer scheme of the turbulent viscosity distribution over the flow cross-section was developed in [20].

According to experimental data, the axial velocity distributions in the boundary zones of the 'Prandtl layer', while remaining logarithmic, significantly deviate from the theoretical values with the canonical choice of constants $\kappa = 0.4, C_r = 5.5$ and $C_{sh} = 8.48$. Variability of these constants is indicated in [21–23]. Assumption about the dependence of these constants on the hydraulic drag coefficient [24, 25] enabled to set more accurate mutually consistent relationships for axial velocities. To do this, the authors [24, 25] suggest that the von Karman constant and the turbulence constants change as follows:

$$C_{sh} = C_r + \frac{1{,}2}{\kappa}, \frac{1}{\kappa} = 1{,}6\left(\frac{1}{\sqrt{\lambda_r}}\right)^{0{,}25}$$

Hypotheses of Prandtl, von Karman and others on the turbulence structure enabled to determine the hydraulic drag coefficient and velocity profiles for a hydraulically smooth wall ($0 \le u_* k_s / \nu \le 5$, $\lambda = \lambda\left(\mathrm{Re}\right)$, $\mathrm{Re} = 2R_0 V / \nu$, V– average volume velocity) and for the flow in a quadratic area ($70 \le u_* k_s / \nu$, $\lambda = \lambda\left(R_0 / k_s\right)$). Between these areas, there is a buffer zone ($5 < u_* k_s / \nu < 70$) that plays an important role in the heat exchange process and is not taken into account by the traditional semi-empirical theory, thus often pushing to use assumptions that contradict reality when determining the integration constant in the logarithmic law equation (2).

Relationships (4), (5) given in the works of various authors, for example [24, 25], do not allow to perform calculations in the transient conditions of turbulent flow. A two-layer approach with a constant mixing path length is considered in [26]. Author [27] showed that the hydraulic drag coefficient in the implicit formulas of Prandtl and Colebrook–White can be explicitly expressed as a Lambert function that depends only on $\kappa / 2R_0$ and Re values. To improve the accuracy of representing the λ relationship, the author [28] proposed to use the correction function f, depending on the dimensionless

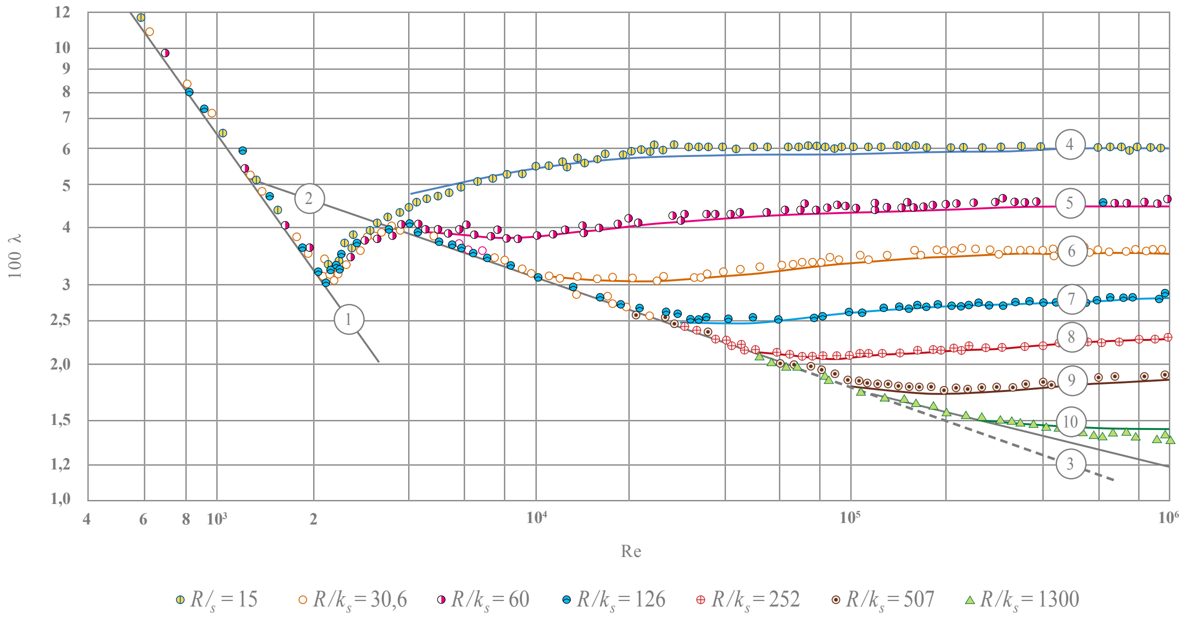

***Figure 1.** Hydraulic drag law for smooth and rough pipes [29]. Curve 1 corresponds to the laminar flow; curve 2 – to Blasius formula; curve 3 – to Prandtl formula. Curves 4–10 – calculations using formula (6).*

number $\kappa u_* / v$, in the Colebrook–White expression for the hydraulic drag coefficient:

$$\frac{1}{\sqrt{\lambda_r}} = 1,74 - 2\lg\left[\frac{k_s}{R_0} + \frac{18,7}{\mathrm{Re}\sqrt{\lambda}}\right],$$

writing it as follows

$$\lambda = \left\{-2\lg\left[\frac{k_s}{R_0 \cdot 7,413} + 2,5226\frac{f}{\mathrm{Re}\sqrt{\lambda}}\right]\right\}^{-2},$$

$$f = 1 - 2.2\lg\frac{k_s u_*}{v}. \quad (6)$$

Comparison of experimental λ values and calculated ones using formula (6) is given in Fig. 1 [29]. The rough surface is considered to be hydraulically smooth according to the condition on equality of λ values, calculated using formulas (4) and (6) [29].

Authors [28, 29] state that most of the methods proposed in the scientific and technical literature for calculating turbulent flow in the transient area are based on the conjugation of computational relationships for hydraulically smooth and extremely rough surfaces.

The consequence of separate consideration of laminar and turbulent liquid flow modes is their separate theories. Regarding this, it's worth noting the approach based on the idea of using a rheological ratio, which is valid for any flow mode [30, 31]. Among few works aimed at solving this problem, authors [32, 33] can be also mentioned. In the cited works, a rheological equation is used that links the equivalent stress tensor τ with strain velocity tensor S as follows:

$$\tau = \frac{2\mu}{1-\tilde{f}}S, \quad S = \frac{1}{2}\left[\nabla\bar{u} + \left(\nabla\bar{u}\right)^T\right], \quad (7)$$

where μ – dynamic viscosity; $\tilde{f}$ – function of coordinates and flow parameters; T – transposition operation.

One shall keep in mind that the Navier – Stokes equations [34] are modified in this case. For the function $\tilde{f}$, the closing transfer equation [35] shall be written, and in the simplest cases it shall be adjusted in order to describe the experimental data in the best possible way [36]. The attractiveness of this approach lies in the possibility to perform calculations based on it, regardless of the flow mode [37].

The difficulties associated with deriving relationships for the velocity profile based on semi-empirical models of turbulence and, consequently, for the hydraulic drag coefficient have led to ongoing attempts to improve purely empirical relationships by making various adjustments to them.

Historically, the search for empirical regularities has followed the separate trends for pipes and for channels. Construction of relationships for the hydraulic drag coefficient, which is included in the formula for pressure loss during flow in pipes, which is often called the Dupuis formula [38] for λ = const –

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$$I = \frac{\lambda}{R_0} \cdot \frac{V^2}{4g} \quad (8)$$

– has followed two trends.

Works in the first trend are characterized by replacing the constant λ by empirically sought functions of R_0, V etc., approximating the calculated values to the experimental data. Works in the second trend seek for changed values (including fractional) of all parameters R_0, V, I included into the Dupuis formula [11] instead of integers –1, 2, 1, while maintaining the constant character of λ.

Probably, the roughness characteristics in the Dupuis formula were first introduced by Darcy [39]. They were later extended to open flows [40]. As a result, the idea was implemented of deriving a generalized relationship, the same for pipe and channel flows, outlined in [41]. Over time, for the value λ a large number of different empirical relationships of the pressure loss I have been proposed, which were based on the approximation of the binomial type belonging to the first trend.

In parallel, the empirical development of monomial calculation formulas with changed parameters R_0, V, I was carried out. The basis of this trend was Reynolds studies. This version laid the foundation for development of many similar monomial fractional-exponential relationships of the type:

$$I = AV^aR^b; \quad R = F/\chi, \quad (9)$$

where F – cross-section flow area; χ – wetted perimeter of the cross-section; A, a, b – constants; R – hydraulic radius of the liquid flow. For flows that take up the entire pipe section, the four-fold hydraulic radius is equal to the diameter ($4R = D$).

According to [42], exponential formulas are particularly suitable for deriving universal formulas that take into account all factors that affect the drag mode. A similar universal formula was given by Reynolds:

$$I = a\left(\frac{\mu}{\rho}\right)^{2-n} V^n d^{n-3}, \quad d = 2R_0,$$

where a is a constant value for all liquids that depends only on the wall type.

Disadvantage of monomial-exponential formulas is that the effect of roughness is concentrated only in the coefficient A. At that, some Reynolds experimental results are not used, but the simplicity of the formula and the convenience of constructing logarithmic-graphic tables are attained. In the studies of the first trend, further attempts were made to derive a universal formula that would combine not only different types of roughness, fluids different in nature (both low-compressible and gaseous), but also the turbulent and laminar flow modes. The most detailed studies was conducted by the German engineer Biel [43]. Biel presented the final version of the formula as follows:

$$\lambda = 0,00942 + \sqrt{\frac{\varepsilon}{d}} + \sqrt{\frac{d}{\tau}} \cdot \frac{1}{\mathrm{Re}}.$$

Parameter ε in this formula characterizes the pipe wall roughness; τ is associated with ε and has the same dimension. The approximate version of the formula (for rough pipes) is:

$$\lambda = 0,00942 + \sqrt{\frac{\varepsilon}{d}} + \frac{3,9}{\mathrm{Re}}\sqrt{\frac{d}{\varepsilon}}.$$

Also, the German engineer Lang in 1917 attempted to derive a universal formula suitable for both flow modes [44]:

$$\lambda = \left(\sqrt{\alpha\frac{V - V_{kp}}{V}} + \sqrt{\frac{64}{\mathrm{Re}}}\right)^2,$$

where V_{kp} – the lowest critical velocity of laminar-to-turbulent flow transition is assumed to be equal to $V_{kp} = 2048\,v/d$; α – roughness parameter with values $\alpha = 0.011 \div 0.012$, for smooth pipes – up to 0.048.

From later relationships of this type, the formula [45, 46] may be given:

$$\lambda_p = 0,11\left[\frac{\alpha + \varepsilon + X^{1,4}}{115X + 1}\right]^{0,25},$$

$$\alpha = \frac{68}{\mathrm{Re}}, \varepsilon = \frac{k}{D}, X = \left(28 \cdot \alpha\right)^{10},$$

where k – equivalent absolute roughness of the pipe inner surface.

The unified universal relationships describing the hydraulic drag with high accuracy over the entire range of Reynolds numbers have not yet been derived. Piecewise-continuous functions, each of which is valid within its own limited sub-range of Reynolds numbers, are the most common. The first trend includes the widely used generalization of the Dupuis formula in the form of the Darcy – Weisbach relationship:

$$h = \lambda\frac{L}{D} \cdot \frac{V^2}{2g}. \quad (10)$$

The second trend includes the version of monomial fractional-exponential relationship of type (9), which is derived from (10), if λ for various sub-regions can be described by unified expression of type:

$$\lambda = \frac{A}{\mathrm{Re}^m}, \quad \mathrm{Re} = \frac{4Q}{\pi D v}, \quad (11)$$

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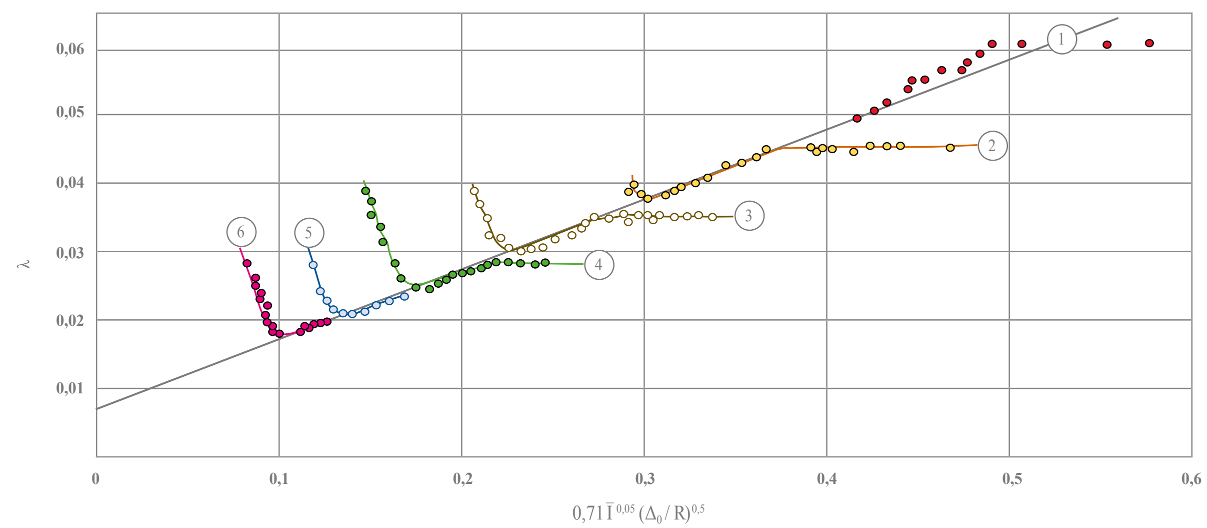

Figure 2. *Summary plot of Nikuradze's experiments regarding flow in pipes with equigranular roughness in coordinates of 'new' link [8]. 1–6 accordingly:* $2R/\Delta_0 = 15, 30, 60, 126, 252, 507; \quad \Delta_0 = 2k_{eq}$.

where A – coefficient in general case depending on roughness; m – constant (its own for every sub-region of the liquid flow); Q – volume flow rate; ν – kinematic viscosity of the pumped liquid.

Substituting (11) into (10), the following relationships are obtained:

$$h = \beta \frac{Q^{2-m} \nu^m}{D^{5-m}} L, \tag{12}$$

$$\beta = \left(\frac{4}{\pi}\right)^{2-m} \cdot \frac{A}{2g}. \tag{13}$$

In the domestic literature, expression (12) is called as the Leibenzon generalized formula. It describes the pressure friction losses versus main factors and is often used for numerical calculations and analytical solutions of some problems.

Solving (13) relative to parameter A using (11), the transition from (12) to (10) is always possible. For the transition from (10) to (12) in relationships included in the first trend (binomial or logarithmic), approximation (11) shall be used. An example of such approximation of Altschul formula with maximum deviation 4.2% is given in works [47, 48]; more accurate approximation – in [49]. The Colebrook-White formula approximation in the form (12) at maximum deviation 4.7% is given in [50].

Currently, both trends have quite extensive set of 'sub-range' formulas [49], the combination of which covers the entire range of changes in the Reynolds number Re, surpassing in accuracy the universal relationships. The error of piecewise-continuous relationships depends on the range to which they belong. In the zone of mixed friction, an error of the Leibenzon formula attains 7% in some versions used [49]. As a result of the comparative analysis, the authors [49] came to the conclusion that the most accurate formulas among the considered are those used in PJSC Transneft (provided that the equivalent roughness keq is determined by solving the inverse problem).

One of the problems in applying piecewise continuous relationships is the abrupt change of the drag coefficient λ value at the border of sub-ranges. A detailed analysis of this problem and possible ways to solve it in relation to practical problems of the fuel and energy complex are given in paper [51], authors of which conclude that such formulas no longer meet modern requirements for accuracy and adequacy of calculations and should be corrected.

Paper [8] gives a sufficiently convincing argument that the choice of the Reynolds number as a stability criterion cannot be considered successful, and more preferable is the universal relative gradient of specific energy

$$\bar{I} = \frac{h_\tau}{L_p} \cdot \frac{gD^3}{64\nu^2}.$$

Meanwhile, to describe the drag coefficient

$$\lambda = \lambda\left(\bar{I}, \frac{k_Э}{R}\right),$$

the following relationship is proposed:

$$\lambda = a + b\bar{I}^x \left(\frac{k_Э}{R}\right)^y, \tag{14}$$

where a, b, x, y – dimensionless experimental constants that depend on the pipe roughness type, flow conditions and drag zone; $k_Э$ – equivalent absolute roughness of the pipe surface.

According to the author [8], formula (14) is '*universal in its ability to generalize mathematically the most diverse sets of experimental (field, laboratory) data in the widest range*'. Applying (14) to the set of Nikuradze's experimental data (364 experiments) regarding flow in round pipes with equigranular roughness made of mineral sand (the experimental basis of the contemporary semi-empirical theory of hydraulic drags), enables to derive the following relationships (three main zones are considered) [8]:

smooth zone

$$\lambda = 0{,}005 + \frac{0{,}138}{\bar{I}^{1/6}}, \bar{I}^{1/2} \frac{k_Э}{R} \le 6,$$

$a = 0{,}005; b = 0{,}138; x = -1/6; y = 0$

transient zone

$$\lambda = 0{,}008 + 0{,}071\bar{I}^{0{,}05} \left(\frac{k_Э}{R}\right)^{1/2}, \quad 6 < \bar{I}^{1/2} \frac{k_Э}{R} < 90,$$

$a = 0{,}008; b = 0{,}071; x = 0{,}05; y = 0{,}5$

square-law zone

$$\lambda = 0{,}008 + 0{,}111\left(\frac{k_Э}{R}\right)^{0{,}4}, \quad \bar{I}^{1/2} \frac{k_Э}{R} \ge 90,$$

$a = 0{,}008, b = 0{,}111, x = 0, y = 0{,}4.$

The summary plot of Nikuradze's experiments regarding flow in pipes with equigranular roughness in coordinates of 'new' link is shown in Fig. 2, where right branches parallel to the X-axis are formed by points belonging to the square-law drag zone. Left branches are formed by experimental points belonging to the 'smooth' drag zone. The given formulas very accurately summarize Nikuradze's experimental data and are adequate to the main formulas of the semi-empirical theory [8]. The accuracy of calculations based on the above formulas is ±2,2%. These ratios (even without taking into account the state of 'technical' roughness) give much better agreement with the experimental data available in the technical literature than, for example, those recommended in the normative documents of PJSC Transneft. Experimental data of Colebrook and White that have led to an interpolation relationship included into current pipeline international standards, can also be generalized using formula (14).

Thus, one may ascertain the following:

- generalization of many empirical studies in pipe hydraulics, conducted for more than a century and a half, enabled to formulate the consistent pattern of pressure loss in the Darcy–Weisbach formula and in its simplified version in the form of monomial fractional-exponential formula, called in the domestic literature the 'generalized Leibenzon formula';
- Reynolds number and relative roughness are accepted as arguments for the hydraulic drag coefficient (an alternative to the Reynolds number is proposed in few works);
- experimentally confirmed theoretical relationships based on Navier–Stokes equations are obtained for the hydraulic drag coefficient under the laminar flow conditions;
- attempts to formulate a closed system of equations for the theory of a viscous liquid turbulent flow were unsuccessful, and the most productive at the moment is the phenomenological approach, based on which the semi-empirical Prandtl–Karman and Colebrook–White relationships were derived for the hydraulic drag coefficient;
- improving the empirical relationships by making various adjustments to them also did not lead to a cardinal success in the accuracy of empirical data presentation, a meaningful result was obtained only in [8].

Of all the considered relationships, formula (14) demonstrated the best generalizing capability both for pressurized flows in pipes and for channel flows with various types of wall roughness, including earth channels. This provide reasons to use the relationship (14) structure for parametric identification of experimental constants included in it in order to define clearly the hydraulic drag coefficient in real pressure pipelines.

The purpose of this work is to apply the relationship proposed in [8] for simulating the hydraulic drag in petroleum products pipelines.

Methods

To model one-dimensional stationary processes, we use the simplest version of a system of equations describing the flow of a viscous poorly compressible liquid in a rigid pipeline with constant cross-section area

$$\frac{d}{dz} Q = 0, Q = \rho u \frac{\pi D^2}{4} = \rho u f, \rho = \rho^o \left(1 + \frac{p - p^o}{K}\right), \tag{15}$$

$$\left(1 + \frac{Q^2}{\rho^o K f^2}\right) \frac{dp}{dz} = -\frac{\lambda |Q| Q}{2D\rho^o f^2} \left(1 - \frac{p - p^o}{K}\right) + $$

$$+\rho^o \left(1 + \frac{p - p^o}{K}\right) g \cos\alpha_z, \tag{16}$$

$$\lambda = a + b\bar{I}^x \left(\frac{k_Э}{R}\right)^y. \tag{17}$$

It should be noted that for the relative gradient of specific energy

$$\bar{I} = \frac{h_\tau}{L_p} \cdot \frac{gD^3}{64v^2}$$

there exists another form of record that indicates the complicated implicit dependence of the parameter λ on Re number:

$$\bar{I} = \frac{\lambda}{128}\mathrm{Re}^2 .$$

At the first stage, we will consider the problem of recovering the parameter λ, which provides agreement with the experimental data. Let $x_1, x_2, \ldots, x_n$ be the pipeline characteristic points, where pressure P_i^{zm} is measured. We consider readings of 'stationary' and 'line' pressure gages as the experimental data, assuming that these readings are filtered by their 'quality'. Parameter λ shall be found by minimizing the functional Ξ (target function) of the root-mean-square deviation of calculated pressure values and measured pressures P_i^{zm}:

$$\Xi = \sum_i \left(P_i^{zm} - p_i\right)^2 .$$

Then, parameter λ shall be found from the condition of functional Ξ minimum:

$$\frac{\partial}{\partial\lambda}\Xi = -2\sum_i \left(P_i^{zm} - p_i\right)\left(\frac{\partial p}{\partial\lambda}\right)_i = 0. \tag{18}$$

Let's introduce the sensitivity function by reference to parameter λ:

$$\frac{\partial p}{\partial \lambda} = \alpha_\lambda .$$

Function α_λ will be determined from equation obtained by differentiating (16) with respect to the parameter λ:

$$\left(1 + \frac{Q^2}{\rho^o K f^2}\right)\frac{d\alpha_\lambda}{dz} = \frac{\alpha_\lambda}{K}\left[\frac{\lambda|Q|Q}{2D\rho^o f^2} + \rho^o g\cos\alpha_z\right] - \frac{|Q|Q}{2D\rho^o f^2}\left(1 - \frac{p - p^o}{K}\right) \tag{19}$$

under initial condition $\alpha_\lambda(0) = 0$.

The joint solution of the system of equations (15), (16), (19) at given value of the parameter λ enables to determine functions p and α_λ simultaneously. Obtained values of p_i and $\alpha_{\lambda i}$ may be used for developing the iteration procedure to recover the parameter λ.

Function $p(z,\lambda)$ in the neighborhood of point $\lambda = \lambda^\kappa$ may be written as:

$$p(z,\lambda) = p(z,\lambda^\kappa) + \alpha_\lambda^\kappa(z)(\lambda - \lambda^\kappa) + O\left((\lambda - \lambda^\kappa)^2\right). \tag{20}$$

From (18) taking into account (20) with accuracy to summands

$O\left((\lambda - \lambda^\kappa)^2\right)$, we obtain:

$$\lambda \approx \lambda^\kappa + \frac{\sum_i \alpha_{\lambda_i}^\kappa \left(P_i^{zm} - p_i\right)}{\sum_i \left(\alpha_{\lambda_i}^\kappa\right)^2}. \tag{21}$$

The approximate value λ, calculated using the formula (21), will be used as the value $\lambda^{\kappa+1}$ in the next iteration:

$$\lambda^{\kappa+1} \approx \lambda^\kappa + \frac{\sum_i \alpha_{\lambda_i}^\kappa \left(P_i^{zm} - p_i\right)}{\sum_i \left(\alpha_{\lambda_i}^\kappa\right)^2}. \tag{22}$$

The condition for terminating the iterative process is selected as:

$$\left|\frac{\lambda^{\kappa+1} - \lambda^\kappa}{\lambda}\right| \le \varepsilon, \lambda \approx \lambda^{\kappa+1}. \tag{23}$$

An identical algorithm, but without defining the drag coefficient structure, was applied in paper [52], where it was noted that such an algorithm is, in fact, a version of the Newton method and has a square-law convergence under condition of a sufficiently good selection of the initial approximation.

At the second stage, we will consider the problem of recovering parameters a,b,x,y, using experimental data and the first-stage calculation information.

Results

Let's introduce sensitivity functions

$$\frac{\partial p}{\partial a} = A_1, \frac{\partial p}{\partial b} = A_2, \frac{\partial p}{\partial x} = A_3, \frac{\partial p}{\partial y} = A_4 .$$

Using, as previously, the functional Ξ minimum conditions, but now by reference to parameters a, b, x, y, taking into account (17), we obtain equations for determining the sensitivity functions:

$$\varphi\frac{dA_1}{dz} = \phi A_1 - \psi, \tag{24}$$

$$\varphi\frac{dA_2}{dz} = \phi A_2 - \psi \bar{I}^x \left(\frac{2k_э}{R_0}\right)^y, \tag{25}$$

Parameter x value			Error			
			$\left\|x - x^0\right\|$		$\left\|x - x^\kappa\right\|$	
x	x⁰	xᵏ	value	%	value	%
0.05	0.04	0.0493	0.01	20	0.007	1.4

Table. *Result of parameter x recovery using the 'calculated pressure data'.*

$$\varphi\frac{dA_3}{dz} = \phi A_3 - \psi(\lambda - a)\ln\bar{I}, \tag{26}$$

$$\varphi\frac{dA_4}{dz} = \phi A_4 - \psi(\lambda - a)\ln\left(\frac{2k_э}{R_0}\right), \tag{27}$$

$$\varphi = 1 + \frac{Q^2}{\rho^o K f^2}, \ \phi = \frac{1}{K}\left[\frac{\lambda|Q|Q}{2D\rho^o f^2} + \rho^o g\cos\alpha_z\right],$$

$$\psi = \frac{|Q|Q}{2D\rho^o f^2}\left(1 - \frac{p - p^o}{K}\right).$$

with initial condition $A_1(0) = A_2(0) = A_3(0) = A_4(0) = 0$.

Since value of λ was determined at the first stage, the system of equations (15), (16), (24) – (27) can be solved at given values of parameters *a, b*. Using similar (22) formula, parameter a can be determined independently from the solution of (16), (24); after that, similarly can be found parameters *x, y,* and then *b*. Selecting a certain sequence in identifying parameters $(\lambda \to a \to x \to y \to b)$ enabled to use the same iterative formula of type (22) without using embedded cycles. To control calculations, formula (17) is used.

Example of calculation to recover the parameter *x*. For the purpose of testing the proposed algorithm, an oil pipeline section was considered with the initial data as follows:

a) conditions – stationary, constant pressure and constant flow rate are maintained at the end of the pipeline

p_L =0.256 MPa at z=L,Q=194.1 kg⁄(s;)

b) pipeline parameters: inner diameter d = 0.509 m, length $L = 1.09 \cdot 10^5$ m, hydraulic drag λ=0.026 (defined at first stage), a = 0.008, b = 0.071, y = 0.5, relative roughness

$k_э / R = 0.008, \varepsilon = 0.01;$

c) parameters of the pumped liquid: density

$\rho^o = 871.13\ \mathrm{kg/m^3}$,

$1/K = 9.487 \cdot 10^{-4}\ \mathrm{MPa^{-1}}$, gravity acceleration $g = 9.81\ \mathrm{m/s^2}$.

When solving the problem, it was assumed that along the length of the pipeline at points $z_1 = L/4$ and $z_2 = 3L/4$, pressure gages are available. 'Field data' P_i^{zm} are taken as values of p_i from preliminary calculation at x = 0.05. Errors were introduced into these 'accurate experimental data' in order to determine the potential capability of the sensitivity functions method to derive the parameter *x*. The problem of identification was solved with initial data p_L =0.256 MPa; Q=194.1 kg/s and with initial approximation of x^0=0,04.

The calculation results given in the Table show the high efficiency of the considered x parameter recovery method.

Discussion

The solution of the inverse problem is usually sensitive to various errors, such as accuracy of mathematical modeling and difference approximation, uncertainty in the value of the input parameters, rounding errors in numerical calculation, and so on. However, it may be difficult to reveal the role of a particular factor in the identification error. Ranking these factors by the degree of influence on the final decision is a separate task that is not considered here. In relation to the leak problem, this issue is partially considered in the paper [53].

In order to identify all the parameters of the accepted relationship for the drag coefficient λ, it is advisable to use retrospective data of pressure measurement in a line section of the pipeline under stationary pumping conditions. Sensors mounted in a line section divide it into a number of segments. In this regard, it is advisable not only to consider the above integral identification of the entire section, but also to identify individually the parameters *a, b, x, y* of the structural relationship (14) in each segment.

Monitoring changes in the hydraulic drag coefficient λ as a whole, and parameters *a, b, x, y* will enable, on the one hand, to correct the process simulation model in a timely manner, and on the other – to get information about factors not taken into account in the adopted model. In the future, it seems appropriate to interpret *a, b, x, y* as diagnostic parameters of the pipeline internal cavity condition and link their dynamics with the individual dynamics of unaccounted factors.

Findings

The chosen identification algorithm is effective in recovering the value of the hydraulic drag coefficient from the data of pressure gages in the line part of the pipeline operating in stationary pumping mode (in a field experiment).

The splitting procedure is fully applicable to the problem of identifying four parameters in the selected relationship for the λ coefficient, which leads to the need to solve five problems for the specified sequence using a unified algorithm without embedded cycles.

The system of equations, which includes the basic flow model of pumping and equations for sensitivity functions, significantly expands the predictive capabilities of hydraulic calculation and allows the formulation of tasks for diagnosing the phase state of the transported fluids.

Competing interests

The author declares that there is no competing interest regarding the publication of this paper.

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