Vol. 2, No 4, December, 2018

Swirling flow of a perfect liquid in modelling washout of tank bottom sediments

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ARTICLE INFO

Received: 17 August, 2018
Revised: 14 September, 2018
Accepted: 8 October, 2018

A B S T R A C T

The vortex-like flow structure desired for washout of the sediment beds can be calculated using an analytical solution to the Euler formula. It is shown that using a combined vortex flow with disturbing the layer of sediment makes it possible to increase the concentration of the suspended particles, and to provide conditions for cleaning.

Key words: spiral vortex, structure of vortex formation, suspended sedimentation

1. Introduction

Spatially localised vortex-like flows of real liquid are interesting with regards to washout of the sediment beds in tanks which are used in the technological process of oil delivery. Transverse circulation parameters for liquid have practical value, as they are supported by technical equipment, as well as the impact of the vortex structure on the distribution of suspended solids. The force of inertia can be seen as prevailing over the force of viscosity during washout, and a model for perfect liquid can be used as a base. It is known that in general it is impossible to create an actual picture of viscous liquid movement using the vortex distribution in a perfect liquid. A solution for a non-viscous liquid cannot substitute solutions for a viscous flow [1]; however, it can be viewed as an approximation, which may serve as a benchmark (as a starting point when studying viscous flows). The swirling flow of a perfect liquid can be examined as an approximate representation of a real localised vortex-like flow, after studies [1, 2]. It is well-known that the internal structure of a vortex, its intensity and size are significantly governing the stability of vortex formation.

In order to analyse the geometry of potential vortex formations while the tank sediment beds washout, we will use a partial analytical solution to the Navier-Stokes equation [3, 4], describing a damped spatially localised vortex-like flow with rotational symmetry for an incompressible liquid with periodic dependence on the velocity field. The Navier-Stokes equations for incompressible viscous liquid in the Gromeka-Lamb form [5], in a cylindrical system of coordinates (r, θ, z), can be presented as follows:

$$
\begin{gathered}
-\frac{\partial H}{\partial r}=\frac{\partial u}{\partial t}+2(\eta w-\zeta v)+4\nu\xi';\ -\frac{1}{r}\frac{\partial H}{\partial \theta}=\frac{\partial v}{\partial t}+2(\zeta u-\xi w)+4\nu\eta' \\
-\frac{\partial H}{\partial z}=\frac{\partial w}{\partial t}+2(\xi v-\eta u)+4\nu\zeta';\ \frac{\partial}{\partial r}\left(ru\right)+\frac{\partial}{\partial \theta}v+\frac{\partial}{\partial z}\left(rw\right)=0; \\
H=W+P+\frac{U^2}{2};\ \frac{\partial}{\partial r}W=-X;\ \frac{1}{r}\frac{\partial}{\partial \theta}W=-Y;\ \frac{\partial}{\partial z}W=-Z \\
\xi=\frac{1}{2r}\left(\frac{\partial w}{\partial \theta}-\frac{\partial rv}{\partial z}\right);\ \eta=\frac{1}{2}\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial r}\right);\ \zeta=\frac{1}{2r}\left(\frac{\partial rv}{\partial r}-\frac{\partial u}{\partial \theta}\right) \\
\xi'=\frac{1}{2r}\left(\frac{\partial \zeta}{\partial \theta}-\frac{\partial r\eta}{\partial z}\right)\ \eta'=\frac{1}{2}\left(\frac{\partial \xi}{\partial z}-\frac{\partial \zeta}{\partial r}\right);\ \zeta'=\frac{1}{2r}\left(\frac{\partial r\eta}{\partial r}-\frac{\partial \xi}{\partial \theta}\right)
\end{gathered}
\tag{1}
$$

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http://doi.org/10.28999/2514-541X-2018-2-4-287-294.

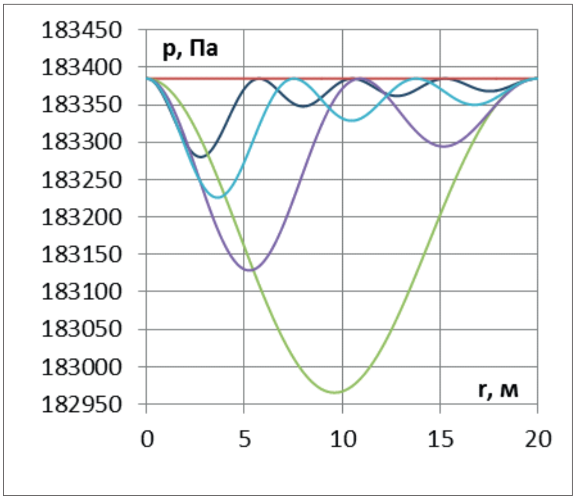

Fig.1. The bottom pressure depending on the number of vortices enclosed.

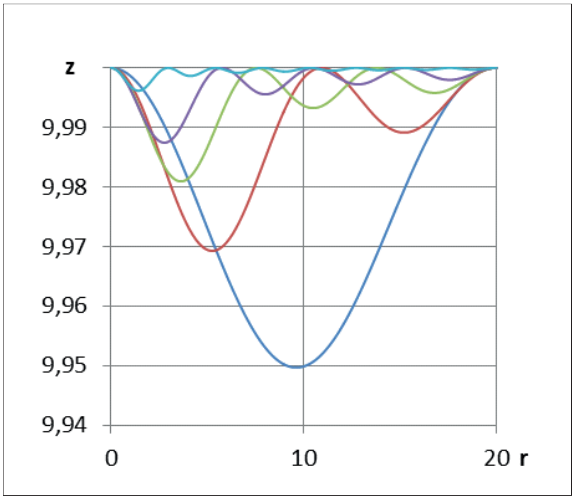

Fig.2. Isobaric surfaces depending on the number of vortices enclosed.

in which:

X, Y, Z are projections of the external forces on the corresponding axes of the cylindrical system of coordinates;
P is the pressure referred to the constant density of the liquid ρ;
U is the module of full speed;
u, v, w are the components of the velocity vector;
ν is the kinematic coefficient of viscosity.

As is well known, for uniform swirling flows of viscous liquid the following formulas are valid:

$$\frac{\xi}{u}=\frac{\eta}{v}=\frac{\zeta}{w}=\frac{k}{2}=\text{const};\quad \xi'=\frac{k^2}{4}u;\quad \eta'=\frac{k^2}{4}v;\quad \zeta'=\frac{k^2}{4}w; \tag{2}$$

Under the condition in Equns 2, Equns 1 are reduced to the form:

$$-\frac{\partial H}{\partial r}=\frac{\partial u}{\partial t}+\nu k^2 u;\quad -\frac{1}{r}\frac{\partial H}{\partial \theta}=\frac{\partial v}{\partial t}+\nu k^2 v;\quad -\frac{\partial H}{\partial z}=\frac{\partial w}{\partial t}+\nu k^2 w \tag{3}$$

A partial solution to Equn 3, examined in Refs 3 and 4, has the form:

$$H=B(t);\quad \mathrm{u}=U_1\exp\left(-\nu k^2 t\right);\quad v=U_2\exp\left(-\nu k^2 t\right);\quad w=U_3\exp\left(-\nu k^2 t\right) \tag{4}$$

where U_i is components of the velocity vector $\boldsymbol{U}$ for a stationary uniform swirling flow of perfect liquid, which can be calculated from the equation:

$$\Delta U+k^2U=0 \tag{5}$$

Given a variable module of full speed $\boldsymbol{U}$ for different points of transverse cross-section of the flow, the pressure will also not be constant. For type (4) flow, the Navier-Stokes equations

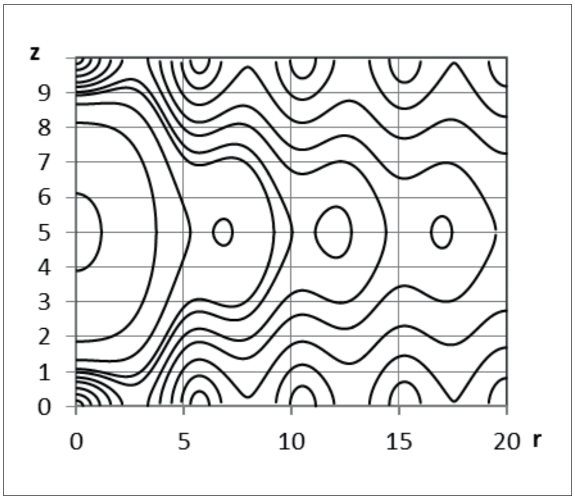

Fig.3. Isolines for the full velocity in the meridian plane.

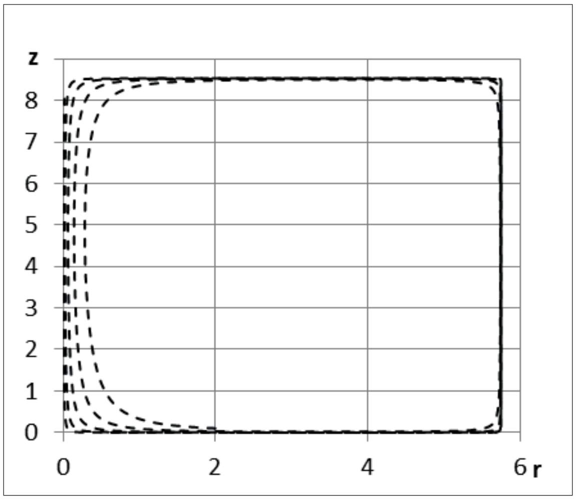

Fig.4. Trajectory of the test particle in the meridian plane of the cell.

externally reduce to the Euler equations, while the impact of viscosity is manifested in the formation of the pressure field. Thus, the kinematic form of swirling motion (velocity field) may be found by means of examining the non-viscous (perfect) liquid.

2. Formulating the problem

The aim of this study is to show the possibility of using analytical solutions to the Euler equations in order to calculate the structural parameters of vortex formation, which causes effective “suspension” and distributes sediment bed particles throughout the volume of liquid. The object of the research is the category of solutions obtained in paper [6].

The conditions for a vortex structure, its geometric dimensions, and the energy necessary for it to form and sustain itself will be interpreted as general requirements for the specifications of a device which can technically implement this. We will examine the rotational-symmetrical stationary movement of a perfect incompressible liquid in the range $l_p \leq z \leq L$ between impermeable planes $z = l_p$, $z = L$ and the impermeable cylindrical surface $r = R$. Plane $z = L$ simulates a pontoon [7] (and, in some cases, a free surface), while plane $z = l_p$ corresponds to the surface of the sediment beds.

The basic structure of a swirling vortex.

The velocity and pressure field for one of the options of partial analytical solution for Equn 5 can be described through relationships [6]:

$$U_r = \frac{n\pi}{kL_p} A J_1(mr)\cdot\cos\frac{n\pi z_p}{L_p}; \quad U_\theta = A J_1(mr)\cdot\sin\frac{n\pi z_p}{L_p}$$

$$U_z = -\frac{m}{k} A J_0(mr)\cdot\sin\frac{n\pi z_p}{L_p}; \quad P = B + g\left(L_p - z_p\right) - 0.5\,U^2;$$

$$z_p = z - l_p; \quad L_p = L - l_p; \quad k^2 = m^2 + n^2\pi^2 \big/ L_p^2 \qquad (6)$$

where:

A, B are the arbitrary constants;

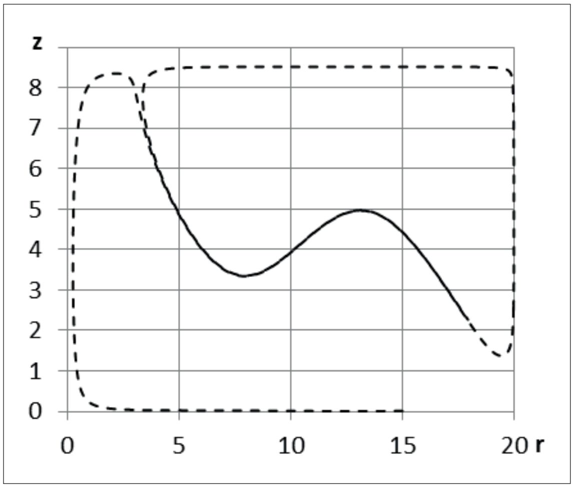

Fig.5. Trajectory of the particle in the meridian section of the composite vortex.

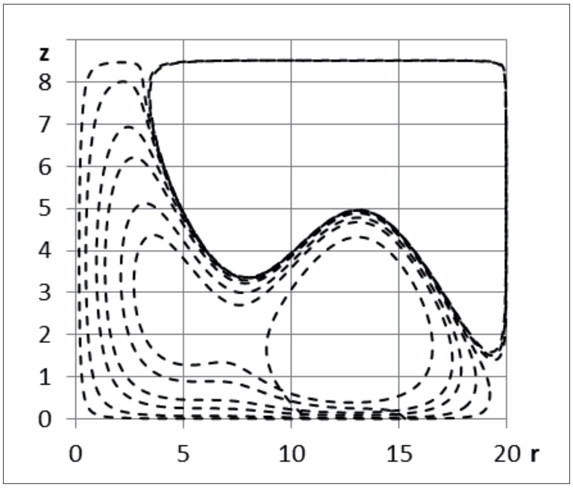

Fig.6. Trajectories of two test particles at vibrations.

J_0, J_1 are the first kind Bessel functions of zero and first order respectively;
l_p is the average height of a sediment bed layer;
g is the free fall acceleration;
n is an integer;
m is one of the roots of the equation $J_1(mR) = 0$.

The vortex described by relationships (6), may form a series of "links" along its length. The length of each link and its mass are unchangeable. Furthermore, it may consist of several (depending on the root number of the equation $J_1(mR) = 0$) vortices (cells) enclosing each other in a radial direction. The real development of the vortex formation examined is associated with reaching kinematic limit conditions - with the formation of a distributed radial flow – diverging in the plane $z = L$ and converging for a liquid layer in contact with sediments $z = l_p$.

The trajectories of movement (6) are obtained based on the definition $dr/U_r = rd\theta/U_\theta = dz/U_z$. In meridian and transverse sections, the respective trajectories have the following form:

$$E = 0.5\rho\int_0^R\int_0^L\int_0^{2\pi} U^2\, r d\theta\, dr dz = 0.5\rho\pi R^2 L_p\left[1+\left(n\pi/L_p m\right)^2\right]U_{*z}^2$$

where C_1, C_2 are arbitrary constants.

In study [8], it was established that a stable aerial vortex between two parallel planes arises and disappears under strictly defined angular velocity (swirling value), depending on the distance between the planes. A similar situation, evidently, can be expected in this case too: when a vortex arises, it shall have a certain minimum reserve of kinetic energy, transmitted to it by an external device. The full kinetic energy of vortex motion with velocity field (Equn 6) can be calculated in the following way:

$$E = 0.5\rho\int_0^R\int_0^L\int_0^{2\pi} U^2\, r d\theta\, dr dz = 0.5\rho\pi R^2 L_p\left[1+\left(n\pi/L_p m\right)^2\right]U_{*z}^2$$

From this, it follows that the critical value of the specific kinetic energy depends on the number of vortices inclosed and the number of their links, as well as on the maximum longitudinal velocity on the tank wall. The critical value of the maximum longitudinal velocity in an external vortex is either determined by the critical kinetic energy, or empirically. A vortex formed in the absence of "external support" will die out. The power of external forces guaranteeing steady-state condition can be assessed from two factors. Firstly, according to the formulas (4), which are justified for a "free vortex":

$$N_{*1} = \nu k^2 \rho \pi R^2 L_p \left[1 + \left(n\pi / L_p m\right)^2\right] U_{*z}^2$$

and secondly, due to the force interaction with the tank walls. The pressure field necessary to describe the force interaction with the tank walls can be described by the equations:

$$P = \frac{p_a}{\rho} - \frac{A^2}{2}\left(b-1\right)J_1^2\left(mR_*\right) + g\left(L_p - z_p\right)$$
$$-\frac{A^2}{2}\left[J_1^2\left(mr\right) + bJ_0^2\left(mr\right)\right] + \frac{A^2}{2}b\left[J_1^2\left(mr\right) + J_0^2\left(mr\right)\right]\cos^2\frac{n\pi}{L_n}z_p$$

$$z_p = z - l_p; \quad b = \left[1 + \left(\frac{n\pi}{mL_p}\right)^2\right]^{-1}$$

Here, it is accepted that at the contour $z_p = L - l_p$, $r = R_*$ the pressure is equal to the atmospheric pressure. The power of drag forces due to the second factor calculated using the Coulomb friction model, takes the following form:

$$N_{*2} = 4n\pi R f \rho \cdot \int_0^l Pw\,\mathrm{d}z = 4f\,R\,L_p\left\{p_a - \rho\, bk^2 w_*^2 / 3m^2 + \rho g L_p\left[\left(2n-1\right)\pi - 2\right] / 2n\pi\right\} w_*$$
$$l = L_p / 2n; \quad w_* = U_{*z}\exp\left(-\nu k^2 t\right); \quad f = \text{const}.$$

The total power output of the technical devices which deliver the flow (6) shall exceed any losses due to the factors indicated above. Moreover, it shall be sufficient to form a primary vortex with the necessary capacity to "suspend" sediment bed particles. All the factors indicated may be assessed within the framework of analytical solutions to the Euler equations. This means that it is possible to formulate the task of choosing a combined vortex with an optimum structure (based on the energy criteria), which would guarantee the cleaning requirements.

In order to model the floating head covers, one shall take $R_* = R$. Figure 1 presents the bottom pressure distribution, depending on the structure of vortex formation given the following choice of initial data: L=10 m; n=1; R_*=20 m; ρ =850 kg/m^3; A=2 m/s; mR=3.83; 7.02; 10.17; 13.32.

According to Fig.1, there are grounds to expect that minimal ridging on the surface of sediment beds will ensure combined vortex formation, including no less than four enclosed vortices. In the area examined in this case, there is no decompression zones where the pressure reaches the saturated vapour pressure. Cross-sections of isobaric surfaces with the meridian plane, presented in Fig. 2, show that at eight enclosed vortices, the plane $z = L$ can practically be considered a free surface at atmospheric pressure. Figure 3 shows isolines for the full velocity of a liquid, giving an idea of isobars for the dynamic pressure component in a cellular vortex consisting of four enclosed vortices. Here, the presence of isolated circular formations is notable, with the plane of symmetry corresponding to the maximum axial velocity.

Parametric calculations for the trajectories of test particles [9] placed at various points in the liquid with velocity field (6) have shown that an accumulation zone forms for suspended particles of paraffin and solids in each cell – the "vortex veil", which is illustrated in Fig.4 for a cell adjoining the axis of symmetry.

It is helpful to use a distribution of suspended particles that is not uniform across the volume of the liquid when arranging the gradual offtake of the slurry (with replacing by the original liquid) in the washout process. When the washout process halts, offtake of a uniform suspension can be performed efficiently. The question arises: how can a uniform concentrated suspension be formed? The composite (cellular) vortex has limited capacity in this sense. Therefore, a combination of vortex and vibration processes seems promising, with the aim of sediment beds flotation and forming concentrated suspensions. We will now preliminarily examine the stability of flow in Equns 4 and 6) in case of small swirling disturbances.

3. Stability

The stability in one of Trkal's non-localised solutions [4] (differing from Equn 6) has been studied in sufficient detail in paper [10]. The case of a localised solution is examined in [11]. According to (4), (6) a partial solution to Equns 1 take the form:

$$u_1 = \exp\left(-\nu k^2 t\right) U_r; \quad u_2 = \exp\left(-\nu k^2 t\right) U_\theta; \quad u_3 = \exp\left(-\nu k^2 t\right) U_z$$
$$P = B(t) - g\left(L_p - z_p\right) - 0.5\, U^2 \exp\left(-2\nu k^2 t\right)$$

Let $u_r = V_r + u_1$; $u_\theta = V_\theta + u_2$; $u_z = V_z + u_3$. Low disturbances of environmental parameters are here notated with subscript letter epsilon (ε). Therefore, the system of equations for the velocity of disturbed flow will take the following form:

$$-\frac{\partial H_\varepsilon}{\partial r} = \frac{\partial V_r}{\partial t} + 2\left[\eta_\varepsilon \left(V_z + u_3\right) - \zeta_\varepsilon \left(V_\theta + u_2\right)\right] + k\left(u_2 V_z - u_3 V_\theta\right) + 4\nu \xi'_\varepsilon$$
$$-\frac{1}{r}\frac{\partial H_\varepsilon}{\partial \theta} = \frac{\partial V_\theta}{\partial t} + 2\left[\zeta_\varepsilon \left(V_r + u_1\right) - \xi_\varepsilon \left(V_z + u_3\right)\right] + k\left(u_3 V_r - u_1 V_z\right) + 4\nu \eta'_\varepsilon$$
$$-\frac{\partial H_\varepsilon}{\partial z} = \frac{\partial V_z}{\partial t} + 2\left[\zeta_\varepsilon \left(V_\theta + u_2\right) - \eta_\varepsilon \left(V_r + u_1\right)\right] + k\left(u_1 V_\theta - u_2 V_r\right) + 4\nu \zeta'_\varepsilon$$

(7)

It is easy to see that a swirling flow will be a partial solution to Equn 7, when there is a common (with a basic flow) eigenvalue for the curl operator:

$$V_r = \varepsilon\left[\frac{n_\varepsilon \pi}{kL_p} J_1\left(m_\varepsilon r\right)\cos\frac{n_\varepsilon \pi}{L_p} z_p\right]\exp\left(-\nu k^2 t\right)$$
$$V_\theta = \varepsilon\left[J_1\left(m_\varepsilon r\right)\sin\frac{n_\varepsilon \pi}{L_p} z_p\right]\exp\left(-\nu k^2 t\right)$$
$$V_z = -\varepsilon\left[\frac{m_\varepsilon}{k} J_0\left(m_\varepsilon r\right)\sin\frac{n_\varepsilon \pi}{L_p} z_p\right]\exp\left(-\nu k^2 t\right); \quad m_\varepsilon^2 + \left(n_\varepsilon \pi / L_p\right)^2 = k^2$$

(8)

This result also follows directly from [6, p. 65], which shows that the solution to the Euler equations in the form of a superposition of flow (Equn 6) and stationary analogue of flow (Equn 8) may be obtained with two different pairs of values (m,n) which at given values k, l, R satisfy the equation $m^2 + \left(n\ \pi/L_p\right)^2 = k^2$.

These two pairs can always be obtained for specific values of L_p and k:

$$L_p = \pi\sqrt{\left(n_\varepsilon^2 - n^2\right)\Big/\left(m^2 - m_\varepsilon^2\right)};\quad k = \sqrt{\left(m^2 n_\varepsilon^2 - m_\varepsilon^2 n^2\right)\Big/\left(n_\varepsilon^2 - n^2\right)}$$

One possible option is a pair consisting of a cellular and a composite vortex. One of them may appear as a result of the impact of swirling disturbance on the second. Insofar as Equn 8 is the exact solution to system (7), then the parameter ε may be replaced with an arbitrary constant. The superposition of these vortices by virtue of “improved stability” is advisable for practical implementation. As an example of such a pair, one could take a cellular vortex consisting of four enclosed vortices and a composite vortex of two links. The height of the liquid layer and the eigenvalue of the curl operator are equal to 8.523 m and 0,761 m^{-1} respectively. The typical trajectory of sediment particles of paraffin is presented in Fig.5 for values of constants A,B = -2 m/s. The suspended particles in this option accumulate in the peripheral zone of the vortex, positioned no less that one metre above the lower plane.

A combined vortex with different eigenvalues for the curl operator is constructed by replacing the flow in the near-axis cell of the random swirling vortex with a flow having a different eigenvalue. At that, interfacing conditions shall be met that include the pressure balance on the interface, which is equivalent to the balance of axis velocities. Taking into account Equn 6, the interfacing condition can be expressed in the form:

$$A_2 = A_1 \frac{m_1}{m_2}\sqrt{\left(\left(n\pi / L_p\right)^2 + m_2^2\right)\Big/\left(\left(n\pi / L_p\right)^2 + m_1^2\right)} \cdot J_0\left(m_1 R_1\right) / J_0\left(m_2 R_1\right)$$

where the subscript 2 relates to the parameters of a vortex with another eigenvalue for the curl operator.

The following choice of parameters is a possible option:

$$m_1 R_1 = m_{*1};\quad m_2 R_1 = m_{*j};\quad j > 1$$

where m_* is are the non-zero root of the equation $J_1(m_*) = 0$ in ascending order; J is a random integer (the number of the non-zero root).

3. The basic flow when vibrations are applied

Leaving aside the mechanism for vibrational processes in the sediment bed layer, let us examine the behaviour of test particles in the velocity field of the basic flow. The impact of vibrations simulates the assignment of the non-zero longitudinal particle velocity at the starting point. Figure 6 presents the results of calculating trajectories, which shows the possibility of forming a uniform slurry of suspended particles in the near-bottom volume of the tank. The disadvantage of this vortex structure is the presence of stagnant zone next to the tank wall, which is a consequence of the total velocity of the liquid equalling zero.

For more effective capture of bottom sediments, it is desirable to prevent rotational motion in planes with zero axis velocity of liquid, and to ensure a non-zero initial velocity of sediment particles in the axial direction.

4. Conclusion

Using one of I. S. Gromeka's analytical solutions for swirling flows in a perfect liquid, limits have been obtained for the structure of a composite vortex, and for the devices which create this type of flow in tanks during bottom sediments washout.

Conflicts of interest

All authors have no conflicts of interest to declare.

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To cite: V.V. Zholobov and S.L. Semin. Swirling flow of a perfect liquid in modelling washout of tank bottom sediments. Pipeline Science and Technology. 2018; 2(4): 287-294. DOI: 10.28999/2514-541X-2018-2-4-287-294.